Chemical Diffusion and Thermal Effects on MHD Stagnation Point Flow of Casson Fluids past a Porous Surface

Hassan Waqas, Sajjad Hussain, Shamila Khalid
Department of Mathematics, Govt College University Faisal Abad (Layyah Campus), Pakistan.
syedhasanwaqas@hotmail.com, sajjadgut@gmail.com.shamilakhalid26@gmail.com

Abstract: A numerical study of chemical diffusion and radiative heat transfer effects on magnetohydrodynamic (MHD) stagnation point flow of Casson fluids due to a porous stretching boundary is considered. The governing model of the problem in the form of the non-linear partial differential equations is converted into ordinary differential form by using appropriate non dimensional function. The resulting model is then solved numerically by using a straight forward coding scheme in Mathematica software 11. The results for velocity, temperature and mass diffusion are computed and presented to elaborate the effects of pertinent parameters involved in the study.

Introduction:

The study of non-Newtonian fluids has a variety of applications in engineering and industry especially in extraction of crude oil from petroleum products. Casson fluid is a non-Newtonian fluid which exhibits yield stress. Human blood can also be treated as a Casson fluid due to the blood cells' chain structure and the substances contained like protein, fibrinogen, rouleaux etc. Hence the Casson fluid has its own importance in scientific as well as in engineering areas. The revolution of the boundary layer behavior of a continuous stretching surface started with Sikiadis [1]. Heat transfer characteristics of Casson fluid flow through an exponentially stretching sheet in presence of porous medium and thermal radiation was discussed by Pramanik [2] and concluded that an increase in the value of Casson parameter suppresses the velocity field. Gnaneswara Reddy [3] investigated an unsteady two-dimensional flow of a non-Newtonian fluid past a stretching surface in presence of thermal radiation and variable thermal conductivity. Khalid et al. [4] studied the unsteady free convection flow of Casson fluid past an oscillating vertical plate with constant wall temperature. Three-dimensional MHD boundary layer flow of Casson nanofluid through a linearly stretching surface with convective boundary condition was depicted by Nadeem et al. [5]. Akbar [6] studied the exact solutions of the magneticfield effect on peristaltic flow of a Casson fluid in an asymmetric channel in presence of crude oil refinement. MHD flow of Casson fluid over a stretching surface in presence of Dufour and Soret effects was analyzed by Hayat et al. [7]. Khalid et al. [8] discussed an unsteady free convection MHD flow of Casson fluid through an oscillating vertical plate embedded in a porous medium with constant wall temperature. The stagnation-point flow of non-Newtonian incompressible Casson fluid past a stretching surface in presence of Dufour and Soret effects was depicted by Kameswarani et al. [9], in this study they showed that shrinking case reduces the velocity boundary layer thickness and enhances the concentration boundary layer thickness. Hussanan et al. [10] investigated the heat transfer and Newtonian fluid analysis on an unsteady boundary layer flow of a Casson fluid through an oscillating vertical plate.The study of stagnation point flow towards a stationary semi infinite wall was first introduced by Hiemenz[11] in a two dimensional case. In his work he reduced the Navier-Stokes equations into non linear ordinary differential equations with the help of similarity transformations. The same problem was extended by Homann [12]. Chiam [13] combined the works of Hiemenz and Crane *i.e.*, the stagnation point flow towards a stretching sheet when the stretching rate of the plate is equal to the strain rate of the stagnation point flow and he found no boundary layer structure near the plate. El-Dab [14] obtained numerical solution of MHD flow of micropolar fluid with heat and mass transfer towards a stagnation point on a vertical plate. In all these attempts, stagnation point flow due to the stretching sheet was analyzed. The boundary layer flow over a shrinking sheet was first investigated by Wang [15]. He dealt with stagnation point flow on a two dimensional shrinking sheet and axi symmetric stagnation point flow on a axisymmetric shrinking sheet. In contrast, Lok *et al*. [16] numerically studied non orthogonal stagnation point flow towards a stretching sheet. In his work, he determined that the obliqueness of a free stream line causes the shifting of the stagnation point towards the incoming flow. Many fluids in industries resemble non-Newtonian behaviour. Non-Newtonian fluids are more appropriate than Newtonian fluids because of their varied industrial applications like petroleum drilling, polymer engineering, certain separation processes, food manufacturing etc. For non-Newtonian fluids, the relationship between stress and the rate of strain is not linear and it is difficult to express all these properties in a single constitutive equation. Casson [17] introduced this model to predict the flow behaviour of pigment oilsuspensions of the printing ink type. Later on, several researchers studied Casson fluid pertaining to different flow situations. The unsteady boundary layer flow and heat transfer of a Casson fluid over a moving flat plate with a parallel free stream was studied by Mustafa *et al*. [18]. The exact solution for boundary layer flow of Casson fluid over a permeable stretching/shrinking sheet with and without external magnetic field was discussed by Bhattacharyya *et al*. [19-20].

1.Mathematical Model:

We considered the flow of incompressible and electrically conducting Casson fluids. The flow is steady and two dimensional due to a permeable sheet. Cartesian co-ordinates are used with sheet lying along x-axis. A uniform magnetic field of strength act along y-axis. The induced magnetic field is negligible. The fluid velocity is $\underline{v} = \underline{v}(u, v, 0)$, temperature is T and C is chemical concentration while U is the velocity, and T_∞ is the temperature, C_∞ the concentration of the free stream.

Under the above assumptions the equations governing the problems are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \left(1+\frac{1}{\beta}\right)\upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\mu_0^2 H_0^{\ 2}}{\rho}u - \left(1+\frac{1}{\beta}\right)\frac{\upsilon}{K}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0(T-T_\infty)}{\rho C_p} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{16\alpha}{3\beta^*\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\sigma H_0^2}{\rho C_p}u^2$$

(3)

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 u}{\partial y^2}-R(C-C_\infty)$$

(4)

Where ρ is density, μ is dynamic viscosity coefficient, σ is the electrical conductivity, k is the thermal conductivity, c_p is the specific heat capacity at constant pressure, υ is kinematic viscosityis the stream temperature, Q_0 is the volumetric rate of heat generation, β is Casson fluid parameter, R_n is the radiation parameter, α and β^* are the Stefan-Boltzmann constant, R is the chemical reaction parameter,

The boundary conditions are:

$v=\pm v_0,\ u=0,\ T=T_w, C=C_w$ at $y=0$ (4)

$u(x)=u_e=ax, v=-ay\ , T\rightarrow T_\infty, C=C_\infty$ at $y\rightarrow\infty$

Where a is constant proportional to the free stream velocity for away from the stretching sheet
Using similarity transformations:
The velocity components are described in terms of the stream function ψ (x, y):

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x},\ \psi(x,y)=x\sqrt{a\upsilon}\,f(\eta),\ \eta=y\sqrt{\frac{a}{\upsilon}},$$

$$u=xaf',\ v=-\sqrt{\upsilon a}\,f,\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\ (5)$$

Equation of continuity (1) is identically satisfied.

Substituting the above appropriate relation in equations (2), (3) and (4), we get

$$(1+\frac{1}{\beta})f'''+ff''-f'^2-Hf'-(1+\frac{1}{\beta})\frac{1}{K'}f'+1=0\ (6)$$

$$(4+3Rn)\theta''+3Rn\Pr(f\theta'+\lambda\theta+Ecf''^2+ME_C f'^2)=0\ (7)$$

$$\phi''+S_c(f\theta'-\alpha\phi)=0\ (8)$$

and the boundary conditions are

$$f'(0)=0,\ f(0)=A,\ \theta(0)=1,\ \varphi(0)=1$$
$$f'(\infty)=1,\ \theta(\infty)=0, \phi(\infty)=0 \qquad (9)$$

Where $P_r = \frac{\mu C_p}{k}$ is the Prandtl number, $E_C = \frac{u_e^2}{C_p(T_W - T_\infty)}$ is Eckert number, $\lambda = \frac{Q_0}{\rho C_p a}$ is the heat source (λ<0) or sink (λ>0), $\mathrm{Re}_m = \mu_e H_0 \sqrt{\frac{\sigma}{\rho a}}$ is the magnetic parameter, $K_p = \frac{aK'}{\upsilon}$ is the porosity parameter,and $A = \pm \frac{v_0}{\sqrt{a\upsilon}}$ is the suction parameter.

3. Results and Discussion:

Finally, the equations of the motion as given above namely equation (6) to equation (9) are required to be solved. These equations are difficult to yield any close form solution. In order to obtain numerical solution of the problem, these higher order equations are reduced to first order ODE's which are then solved numerically with ND Solved command in Mathematica version 11.Relaible results for velocity and temperature and mass diffusion functions have been computed for sufficient ranges of the physical parameters namely magnetic field parameter H_a**,** Casson flux parameter β**,** porosity parameter K**,** Prandtl number P_r**,** Radiation parameter R_n**,** heat source parameter λ and Eckert number E_c**.** The results are presented in the form of plots for velocity and temperature distributions.

It is noticed that Casson parameter β cause increase in horizontal velocity component f' shown in fig.1.But increase in magnetic field strength and suction at the surface, both decrease the velocity as depicted respectively in fig.2 and fig.3.

Fig.4 shows that the velocity decreases with increase in the value of porosity parameter *K*.

The temperature function $\theta(\eta)$ increase in magnetic with increase in the values of magnetic field H_a, radiation parameter R_n and Eckert number E_c as presented respectively fig.5, fig.6 and fig.7. But the Prandtl number P_r reduced the magnitude of temperature distribution as shown in fig.8.

The effect of heat source and sink parameter λ is demonstrated in fig.9. The temperature increase with increase in λ(λ>0) and it decrease when λ<0. The increase in the value of suction parameter causes increase in temperature distribution as depicted in fig.10.

Fig.11 shows the mass diffusion $\theta(\eta)$ decreases with increase in the values of Schmidt number S_c. But the mass diffusion $\theta(\eta)$ increases with increase in the values of solutal parameter α as shown in fig.12.

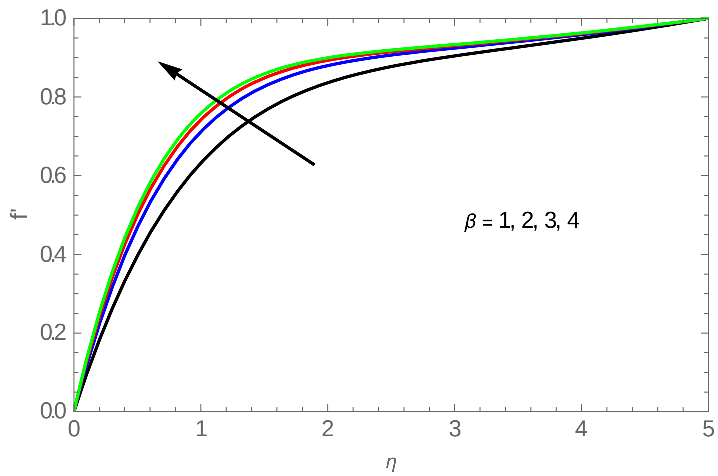

Fig.1: The plot for curves of f' under the effect of various values of β λ=0.1,

P_r=0.7, E_c=0.1, and R_n=0.1.

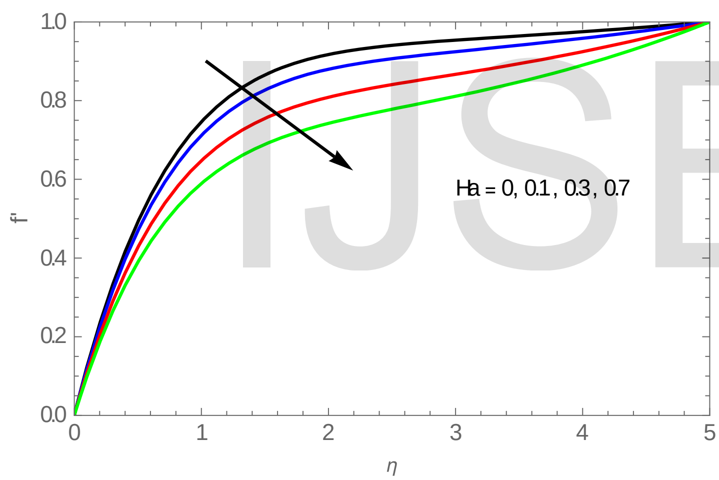

Fig.2: The plot for curves of f' under the effect of magnetic parameter H_a when A=1, P_r=0.7, E_c=0.1, $\beta=1$, λ=0.1 and R_n=0.1

ISSN 2229-5518

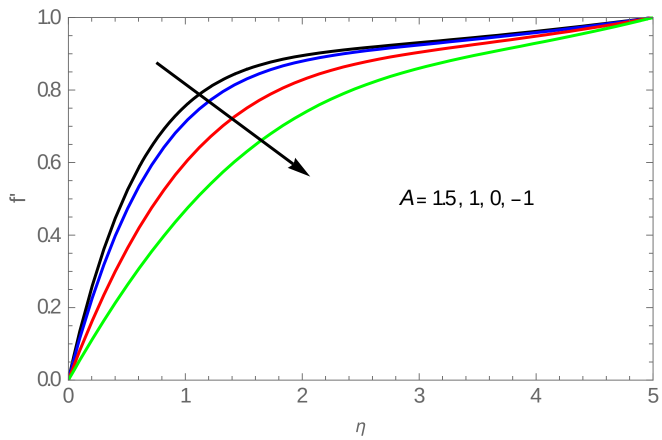

Fig.3: The plot for curves of f' effect of Suction parameter A when λ=0.1, H_a =2 P_r=0.7, E_c=0.1, and R_n=0.1.

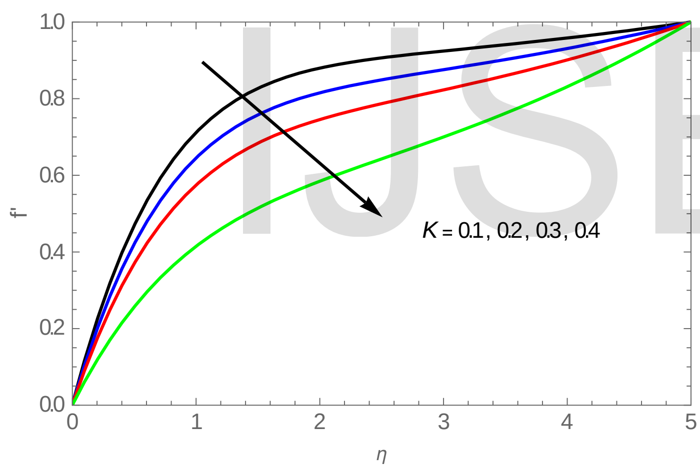

Fig.4 The plot for curves of f' under the effect porosity parameter K, λ=0.1, H_a =2, A=1, P_r=0.7, E_c=0.1, and R_n=0.1.

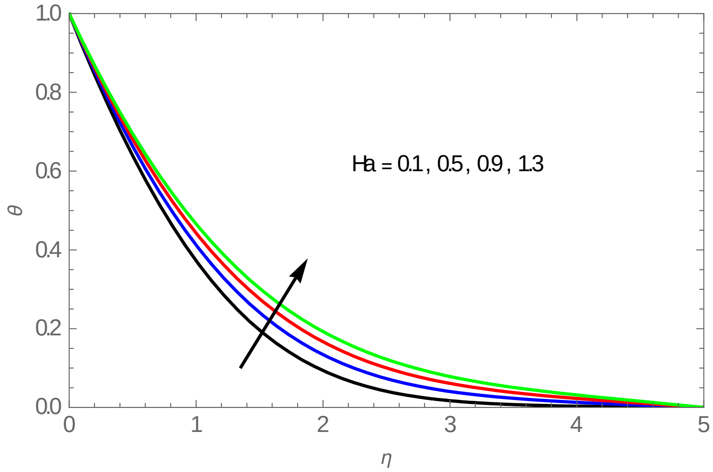

Fig.5: The plot for curves of θ under the effect of Magnetic parameter H_a when, A=1, E_c=0.1, and P_r=0.7

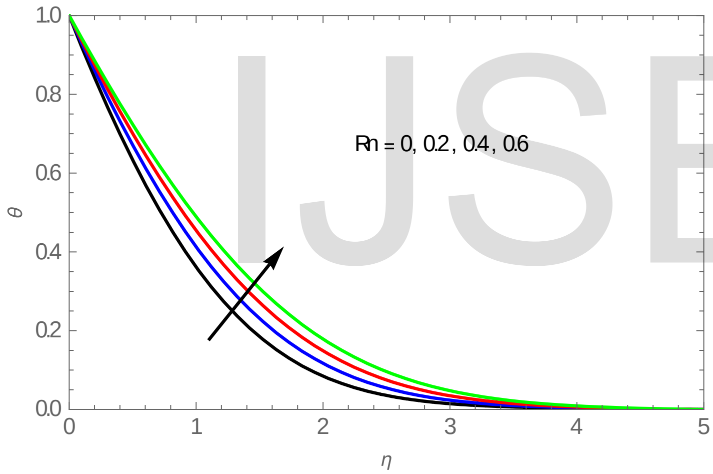

Fig.6: The plot for curves of θ under the effect of radiation parameter R_n when λ=0.1, H_a =2, A=1, P_r=0.7, E_c=0.1, and R_n=0.1.

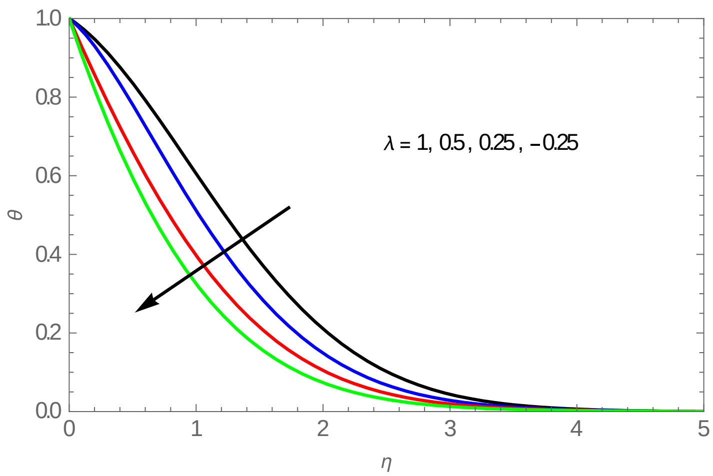

Fig.7: The plot for curves of θ under the effect of heat source parameter λ when H_a =2, A=2, P_r=0.7, E_c=0.1 and R_n=0.1.

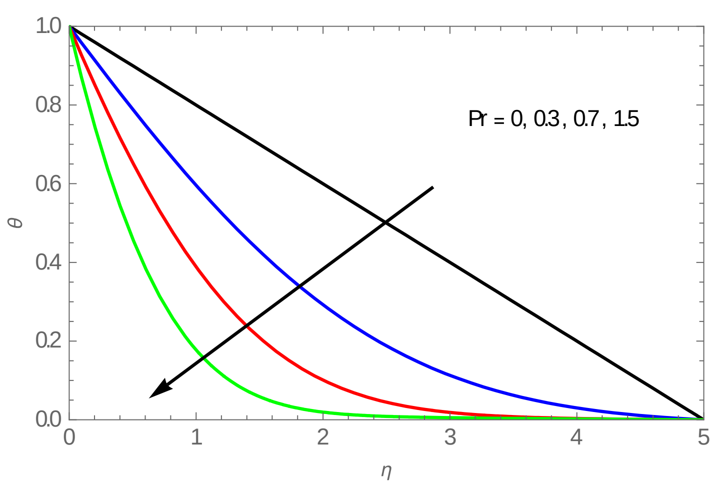

Fig.8: The plot for curves of θ under the effect of Prandtl number P_r when, H_a =2, A=1, P_r=0.7, E_c=0.1 and R_n=0.1.

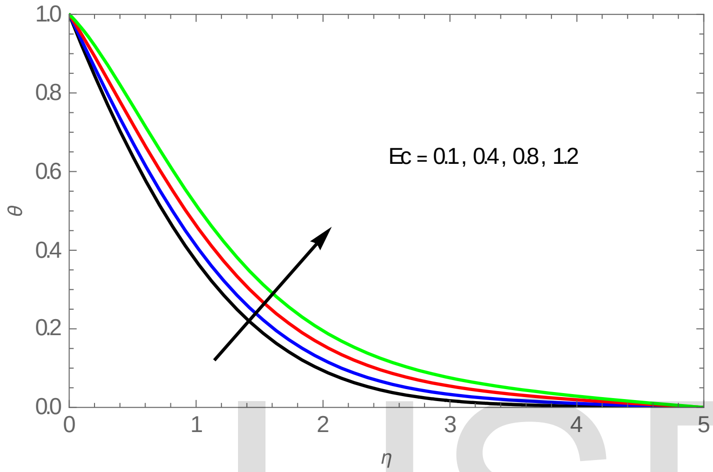

Fig.9: The plot for curves of θ under the effect of Eckert number E_c when λ=0.1, H_a =2, A=1, P_r=0.7, E_c=0.1 ,and R_n=0.1.

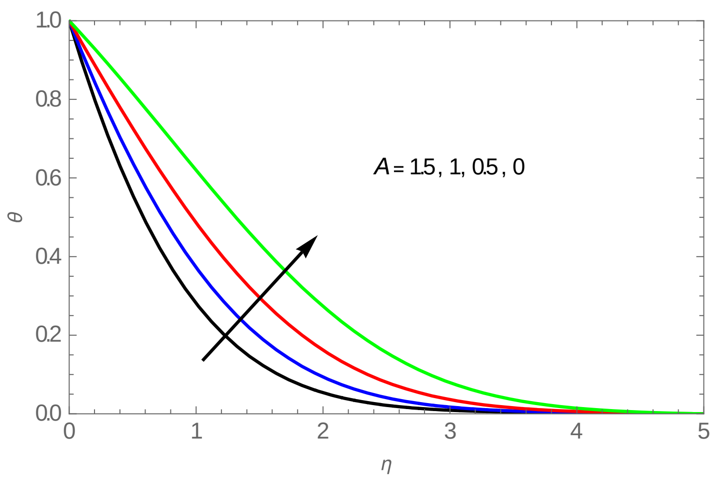

Fig.10: The plot for curves of θ under the effect of suction parameter when λ=0.1, H_a =2 P_r=0.7, E_c=0.1 and R_n=0.1.

ISSN 2229-5518

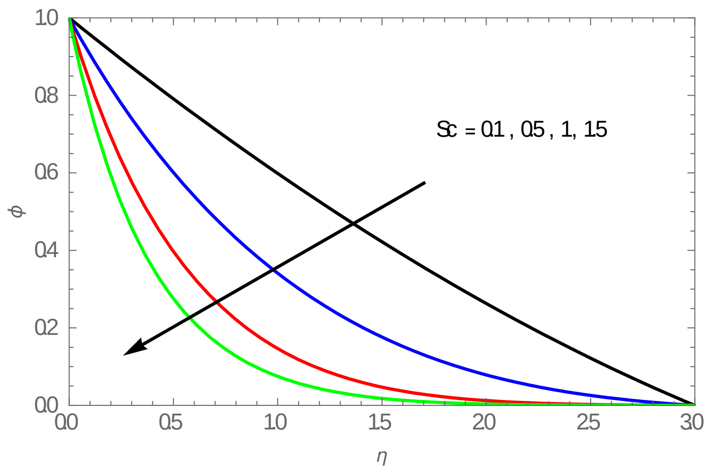

Fig.11: The plot for curves of ϕ under the effect of Schmidt parameter S_c when λ=0.1, A=1, P_r=0.7, E_c=0.1 and R_n=0.1.

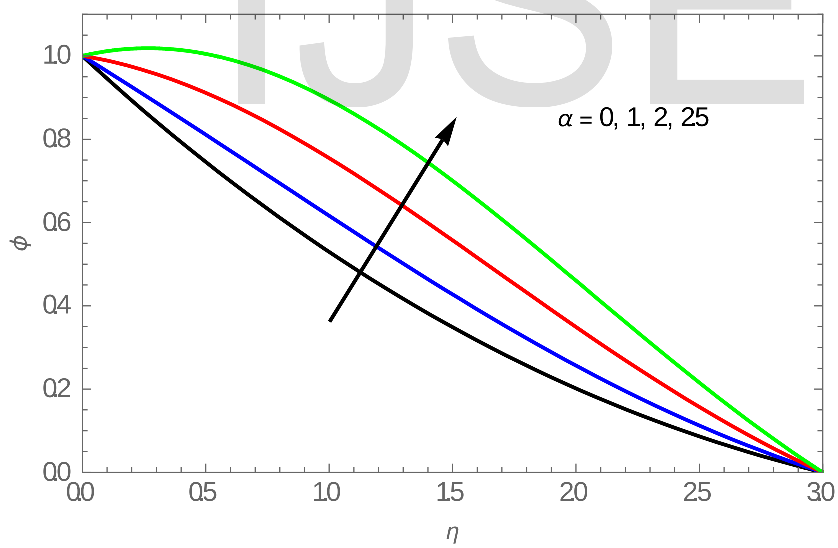

Fig.12: The plot for curves of ϕ under the effect of α λ=0.1, S=2, P_r=0.7, E_c=0.1 and R_n=0.1.

References:

[1] B.C. Sikiadis, Flow past a continuouslymoving plate, A. I. Ch. E. J. 7 (1961) 26–32.

[2] S. Pramanik, Casson fluid flow and heat transfer past an exponentially porous stretching sheet in presence of thermal radiation, Ain Shams Eng. J. 5 (2014) 205–212.

[3] M. Gnaneswara Reddy, Unsteady radiative-convective boundary layer flow of a Casson fluid with variable thermal conductivity, J. Eng. Phys. Thermo Phys. 88 (1) (2015) 240–251.

[4] A. Khalid, I. Khan, S. Shafiel, Exact solutions for unsteady free convection flow of a Casson fluid over an oscillating vertical plate with constant wall temperature, Abstr. Appl. Anal. (15) (2015) 946350 <http://dx.doi.org/10.1155/ 2015/946350>.

[5] S. Nadeem, R.L. Haq, N.S. Akbar, MHD three-dimensional boundary layer flow of Casson nanofluid past a linearly stretching sheet with convective boundary conditions, IEEE Trans. Nanotechnol. 13 (1) (2014) 109–115, doi:10.1109/ TNANO.2013.2293735.

[6] N. Akbar, Influence of magnetic field on peristaltic flow of a Casson fluid in an asymmetric channel: application in crude oil refinement, J. Magnet. Magnet. Mater. 378 (2015) 463–468.

[7] T. Hayat, S.A. Shehzad, A. Alsaedi, Soret and Dufour effects on magnetohydrodynamic (MHD) flow of Casson fluid, Appl. Math. Mech. 33 (10) (2012) 1301–1312.

[8] A. Khalid, I. Khan, A. Khan, S. Shafie, Unsteady MHD free convection flow of Casson fluid past over an oscillating vertical plate embedded in a porous medium, Eng. Sci. Technol. Int. J. (2015) doi:10.1016/j.jestch.2014.12.006 (inpress).

[9] P.K. Kameswarani, S. Shaw, P. Sibanda, Dual solutions of Casson fluid flow overa stretching or shrinking sheet, Sadhana 39 (6) (2014) 1573–1583.

[10] A. Hussanan, M. Zuki Salleh, R.M. Tahar, I. Khan, Unsteady boundary layer flow and heat transfer of a Casson fluid past an oscillating vertical plate with Newtonian heating, PLoS ONE 9 (10) (2014) e108763, doi:10.1371/ journal.pone.0108763.

[11] Hiemenz, K. (1911) Die Grenzschicht an einem in den gleichförmigen Flüssigkeitsstrom eingetauchten geraden Kreiszylinder. Dinglers Polytech Journal, 326, 321-410.

[12] Homann, F. (1936) Der Einfluss grosset Zähigkeit bei der Strömung um den Zylinder and um die Kugel. Zeitschrift für Angewandte Mathematik und Mechanik, 16, 153-164.

[13] Chiam, T.C. (1994) Stagnation-point flow towards a stretching plate. Journal of the Physical Society of Japan, 63, 2443-2444. http://dx.doi.org/10.1143/JPSJ.63.2443

[14] El-Dab, N.T., Ghaly, A.Y., Rizkallah, R.R., Ewis, K.M. and Al-Bareda, A.S. (2015) Numerical solution of MHD Flow of Micropolar fluid with heat and mass transfer towards a stagnation point on a vertical plate. American Journal of Computational Mathematics, 5, 158-174. http://dx.doi.org/10.4236/ajcm.2015.52013

[15] Wang, C.Y. (2008) Stagnation Point Flow towards a Shrinking Sheet. International Journal of Non-Linear Mechanics, 43, 377-382. http://dx.doi.org/10.1016/j.ijnonlinmec.2007.12.021

[16] Lok, Y.Y., Amin, N. and Pop, I. (2006) Non-orthogonal stagnation point flow towards a stretching sheet. International Journal of Non-Linear Mechanics, 41, 622-627.

[17] Casson, N. (1959) A Flow Equation for Pigment Oil Suspensions of the Printing Ink Type. In: Mill, C.C., Ed., Rheology of Disperse Systems, Pergamon Press, Oxford, 84-102.

[18] Mustafa, M., Hayat, T., Pop, I. and Aziz, A. (2011) Unsteady boundary layer flow of a casson fluid due to an impulsively started moving flat plate. Heat Transfer, 40, 563-576. http://dx.doi.org/10.1002/htj.20358

[19] Bhattacharyya, K., Hayat, T. and Alsaedi, A. (2014) Exact Solution for Boundary Layer Low of Casson Fluid over a Permeable Stretching/Shrinking Sheet. Zeitschrit fur Angewandte Mathematik und Mechanik, 94, 522-528. http://dx.doi.org/10.1002/zamm.201200031

[20] Bhattacharyya, K., Hayat, T. and Alsaedi, A. (2013) Analytic solution for magneto hydrodynamic boundary layer low of casson fluid over a stretching/shrinking sheet with wall mass transfer. Chinese Physics B, 22, Article ID: 024702. http://dx.doi.org/10.1088/1674-1056/22/2/024702